



ECF22 - Loading and Environmental effects on Structural Integrity

## Mark tracking technique for experimental determination of fracture parameters

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### Abstract

The Mark Tracking method represents a modern and simple optical method to determine the displacement and strain fields. The principle of the marks tracking method is based on the comparison between two images acquired before and after sample deformation. The algorithm of method track the local displacement of the marks in two directions. Having measured displacements in certain points around the crack it is relative simple to estimate the Stress Intensity Factor (SIF) by displacement correlation methods. The students can compare different data processing algorithms for extracting the SIF's. Basically, a simple geometry like Single Edge Notch Bend specimen loaded in Three Point Bending was tested, for which the exact solution of SIF is well known. The reliability of experimental technique is assessed by comparing the results with the exact solution.

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*Keywords:* mark tracking, displacement correlation, stress intensity factor

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### 1. Introduction

Fracture parameters like Stress Intensity Factors (SIF), Energy Release Rate (ERR), J-Integrals (J) and Crack Tip Opening Displacement (CTOD) are basic parameters in structural integrity assessment based on Fracture Mechanics concept, Saouma (2007). For linear elastic materials the fracture criterion based on Stress Intensity Factors  $K$  and fracture toughness  $K_{IC}$  is the common used:  $K < K_{IC}$ . In order to apply this criteria two sets of data are required: the value of  $K$  which characterize the stress singularity at the crack tip and depends on applied load, crack length,

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geometry of cracked body and loading type. The values of SIF's could be determined analytically, numerically or experimentally, Fig. 1.

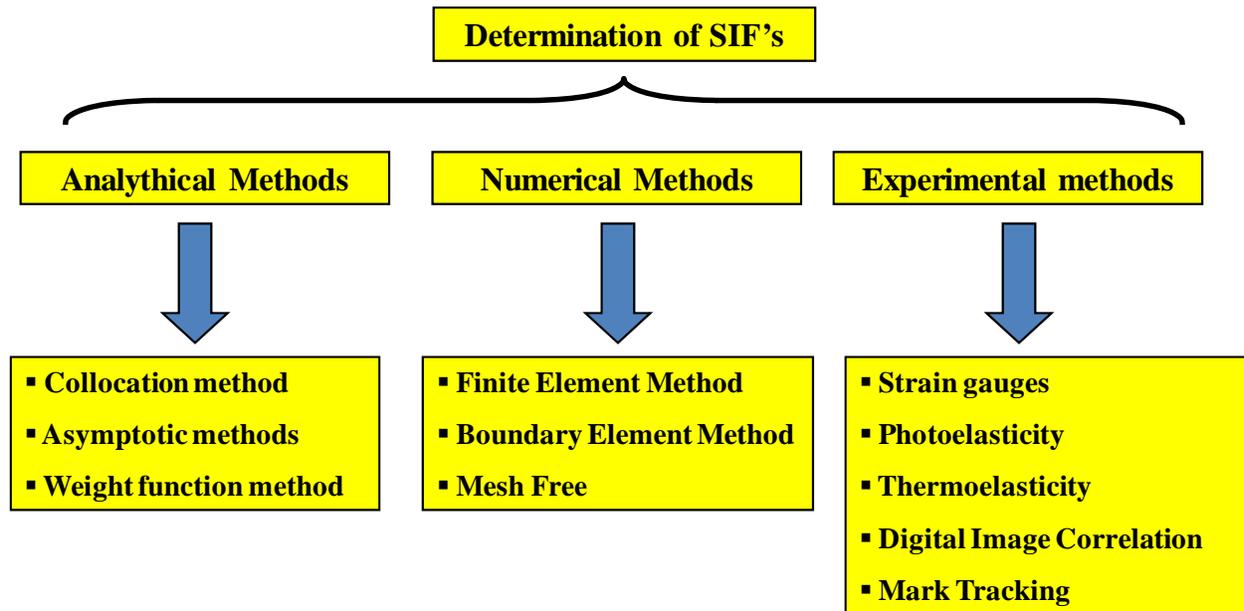


Fig.1 Stress Intensity Factors determination methods

First attempts to introduce SIF's for different types of cracked bodies were performed analytically using collocation method, asymptotic methods, weight function methods, singular integrals. These solutions of SIF's were archived in handbooks, like Tada et al. (1985), Murakami (19987). The numerical determination of SIF's are reviewed in Ingraffea & Wawrzynek (2003) and Kuna (2010). One of the key issues is to extract the SIF's from the numerical analysis results (displacements, strains, stresses, energy), based on displacement correlation, J-integral, modified crack closure methods.

Almost all experimental stress analysis techniques were adopted to estimate the SIF's for cracked bodies.

This paper presents a simple experimental technique the mark tracking combined to displacement correlation as a simple methodology for teaching students how to find the SIF's for cracked bodies.

#### Nomenclature

a	crack length
B	specimen thickness
$K_I, K_{II}$	mode I and mode II stress intensity factors
P	applied load
r	polar radius
S	span
W	specimen width
u, v, w	displacements
$\theta$	polar angle
$\mu$	shear modulus
$\nu$	Poisson ratio

## 2. Displacement Correlation Methods

The Displacement Correlation (DC) methods are simple and easy to extract SIF's from a Finite Element Analysis (FEA) of a cracked body, Ingraffea & Wawrzynek (2003), or from experimental data. The nodal displacements represent primary FEA results and during years different approaches were propose to estimate the SIF's from the displacement field. Chan et.al. (1970) considering plane strain assumption and triangular elements at the crack tip, Fig. 2.a proposed the evaluation of SIF's with the relations:

$$K_I = \frac{\mu\sqrt{\pi}(v_b - v_a)}{\sqrt{2r}(1-\nu)}, K_{II} = \frac{\mu\sqrt{\pi}(u_b - u_a)}{\sqrt{2r}(1-\nu)}, K_{III} = \frac{\mu\sqrt{\pi}(w_b - w_a)}{\sqrt{2r}} \quad (1)$$

with  $\mu$  the shear modulus,  $\nu$  the Poisson ratio,  $r$  the distance from the crack tip to the corresponding node and  $u_i, v_i, w_i$  are the nodal displacements of point  $i$ , measured on  $x, y, z$  axis. For the case of plain stress  $\nu$  is replaced by  $\nu/(1+\nu)$ .

Most accurate results could be obtained using quarter point crack tip elements, Fig.2. b.

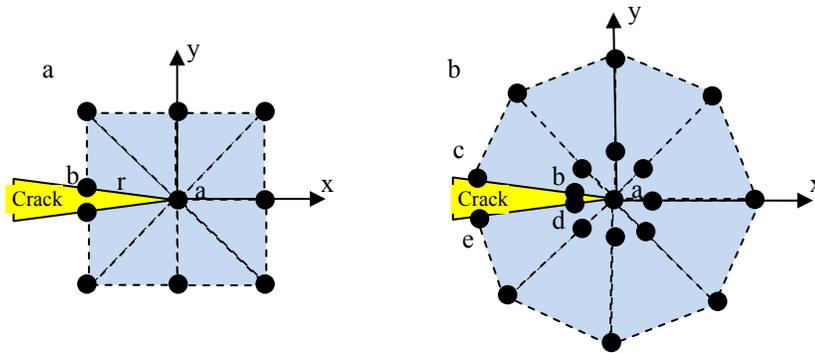


Fig. 2. (a) triangular elements; (b) quarter point crack tip elements.

The SIF's expressions according with Shih et al. (1976) and Tracey (1977) are:

$$K_I = \frac{\mu\sqrt{\pi}[4(v_b - v_d) + v_e - v_c]}{\sqrt{2r}(1-\nu)}, K_{II} = \frac{\mu\sqrt{\pi}[4(u_b - u_d) + u_e - u_c]}{\sqrt{2r}(1-\nu)} \quad (2)$$

The use of displacement correlation technique is simply and allow the separation of different mode of fracture. However, to obtain accurate results a highly refine mesh at the crack tip is necessary and care in selecting the nodes from crack singularity zone. Another alternative is to compute an apparent stress intensity factor  $K_{AP}$  for a series of points approaching the crack tip, to interpolate the results and then extrapolate at the crack tip for  $r \rightarrow 0$ . The processing of data is based on the displacement field near crack tip for a mixed mode load:

$$u = \frac{K_I}{4\mu} \sqrt{\frac{r}{2\pi}} \left[ (2\kappa - 1) \cos \frac{\theta}{2} - \cos \frac{3\theta}{2} \right] - \frac{K_{II}}{4\mu} \sqrt{\frac{r}{2\pi}} \left[ (2\kappa + 3) \sin \frac{\theta}{2} + \sin \frac{3\theta}{2} \right] \quad (3)$$

$$v = \frac{K_I}{4\mu} \sqrt{\frac{r}{2\pi}} \left[ (2\kappa + 1) \sin \frac{\theta}{2} - \sin \frac{3\theta}{2} \right] + \frac{K_{II}}{4\mu} \sqrt{\frac{r}{2\pi}} \left[ (2\kappa - 3) \cos \frac{\theta}{2} + \cos \frac{3\theta}{2} \right]$$

with  $(r, \theta)$  the polar coordinates,  $\mu$  the shear modulus,  $\nu$  the Poisson ratio and  $\kappa=(3-\nu)/(1+\nu)$  for plane stress, respectively  $\kappa=3-4\nu$  for plane strain.

In order to practically apply this technique the displacements on a certain direction  $\theta$ , along a radial line for different distances  $r$  (Fig 3.a) are selected and the apparent SIF's are calculated:

$$\begin{aligned}
 K_I & \begin{cases} (2\kappa - 1)\cos\frac{\theta}{2} - \cos\frac{3\theta}{2} \\ (2\kappa - 1)\sin\frac{\theta}{2} - \sin\frac{3\theta}{2} \end{cases} = 4\mu\sqrt{\frac{2\pi}{r}} \begin{Bmatrix} u \\ v \end{Bmatrix} \\
 K_{II} & \begin{cases} -(2\kappa + 3)\sin\frac{\theta}{2} - \sin\frac{3\theta}{2} \\ (2\kappa - 3)\cos\frac{\theta}{2} + \cos\frac{3\theta}{2} \end{cases} = 4\mu\sqrt{\frac{2\pi}{r}} \begin{Bmatrix} u \\ v \end{Bmatrix}
 \end{aligned}
 \tag{4}$$

Then an interpolation is performed and the value of SIF's is obtained by extrapolation at the crack tip ( $r \rightarrow 0$ ), Fig.3.b.

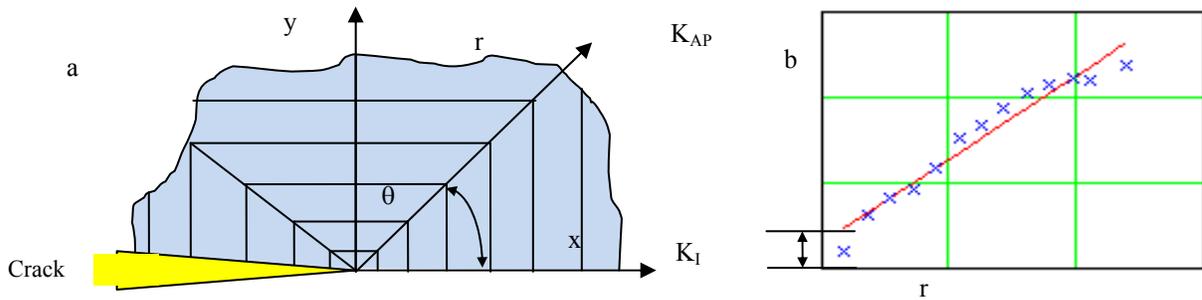


Fig. 3. (a) nodes collected on r direction; (b) extrapolation of apparent SIF at the crack tip.

### 3. Mark Tracking (MT) Techniques

The principle of the method is based on the comparison between two images acquired during the test, before and after deformation Bretagne et al. (2005), Barranger et al. (2013), Pop O. et al. (2013). By comparing the two images, the algorithm of method detects the local displacements in x and y-directions. The principle of the method summarized in Fig. 4. In fact, the displacement of each mark is the translation vector ( $u_1, u_2=v$ ) in  $x_1$  and  $x_2=y$  directions of the centre of gravity.

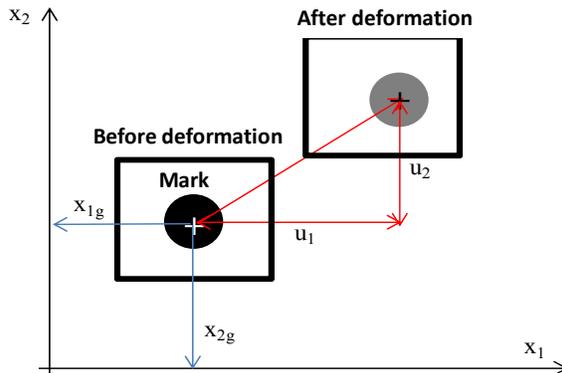


Fig. 4. Principle of Mark Tracking Techniques

According with the principle of method, for the experimental measurements several black marks are positioned on the surface of sample. In our study the marks tracking method allows too definite limits of study zone. The black marks are positioned manually on the specimen surface. Moreover, in order to have a good contrast the black marks are positioned on the white background painted on the specimen surface.

It should be also mentioned that this technique can be usually coupled with the finite element method. In fact, the mark positioned on the surface of specimen may be associated with the nodes of the finite element mesh. As shown in Fig. 4, using the mark positioned on the surface of our sample it is possible to create the meshes with the triangular or the quarter point crack tip elements.

#### 4. Experimental determination of SIF's

A relatively simple procedure to experimentally determine the SIF's is to combine the Mark Tracking technique, which allows the determination of displacements, with the DC method for estimating the SIF's.

A Single Edge Notch Bend (SENB) specimen was employed for experimental tests loaded in Three Point Bending. The specimen is made of rigid polyurethane foam with a density of 145 kg/m<sup>3</sup>, used in civil engineering for thermal isolation. The elastic and mechanical properties of this material are presented in Marsavina et al. (2014, 2015) with a Poisson ratio  $\nu=0.302$ , shear modulus  $\mu=40.62$  MPa, tensile strength  $\sigma=1.87$  MPa and mode I fracture toughness  $K_{IC}=0.131$  MPa m<sup>0.5</sup>. The analytical solution for the mode I SIF is provided in ASTM D5045 (1999):

$$K_I = \frac{P}{BW^{1/2}} f_I\left(\frac{a}{W}\right) \tag{5}$$

$$f_I\left(\frac{a}{W}\right) = 6\left(\frac{a}{W}\right)^{1/2} \frac{1.99 - \left(\frac{a}{W}\right)\left(1 - \frac{a}{W}\right)\left[2.15 - 3.93\frac{a}{W} + 2.7\left(\frac{a}{W}\right)^2\right]}{\left[1 + 2\left(\frac{a}{W}\right)\right]\left(1 - \frac{a}{W}\right)^{3/2}} \tag{6}$$

with  $P$  load,  $B$  specimen thickness,  $W$  width of the specimen,  $a$  crack length and  $f_I(a/W)$  non-dimensional stress intensity factor depending on the crack length. For  $a=19.3$  mm,  $W=50.2$  mm,  $B=25.5$  mm and applied load of 100 N the mode I SIF is  $K_{I,an}=4.198$  MPa mm<sup>0.5</sup>.

On the specimen surface marks were drawn on radial directions from crack tip with a special marker, Fig. 5. The test was performed at room temperature, in displacement control, with a loading speed of 0.5 mm/s, up to 100 N. This load value is lower than the fracture load, and on the linear part of the load-displacement curve.

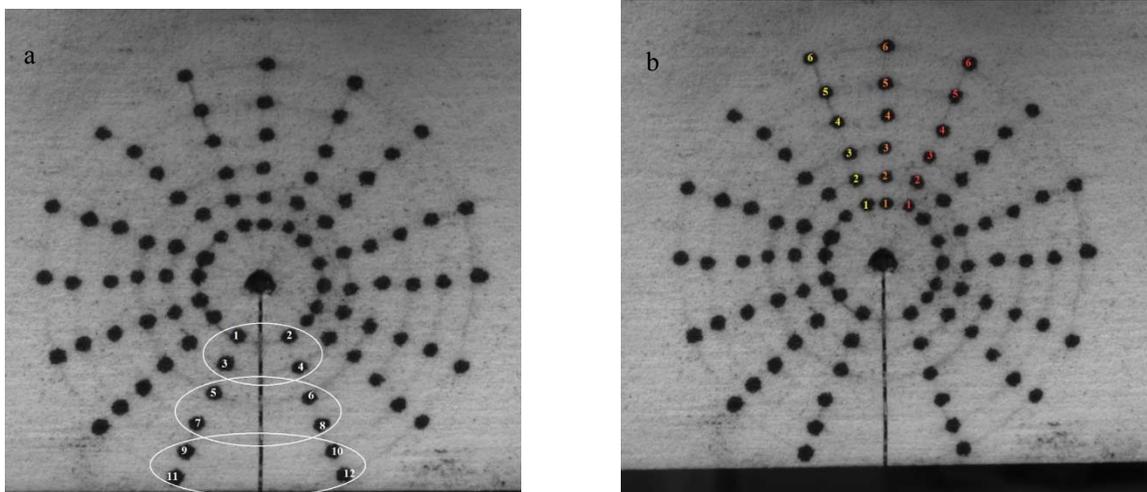


Fig. 5. Marks used for SIF valuation (a) eq. (2), (b) eq. (4)

A USB uEye SE CMOS camera (resolution: 3840 x 2748) with a Pentax 12.5–75 mm lens and a LED light source was used to record 2 fps during the test. In the present study, the Mark Tracking method was performed by using Deftac developed by PEM team of Pprim Institut of Poitiers.

Five combinations of marks were considered (like 1-2-3-4), the displacements of each mark were measured and rel. (2) was applied to determine SIF's, Fig.5.a. A comparison of results with the analytical value  $K_{I,an}$  is presented in Fig. 6. It could be observed that the closest values of the  $K_I$  comparing with the analytical value (green line in Fig. 6) were obtained for the first two mark combinations 1-2-3-4 and 3-4-5-6. This is in agreement with other experimental techniques, like photoelasticity and thermoelasticity, which indicates that the stress singularity zone is between 0.2 to 0.5 crack length (red lines in Fig. 6).

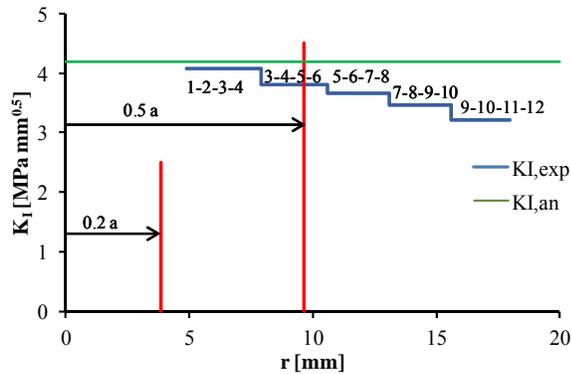


Fig. 6. SIF results based on eq. (2)

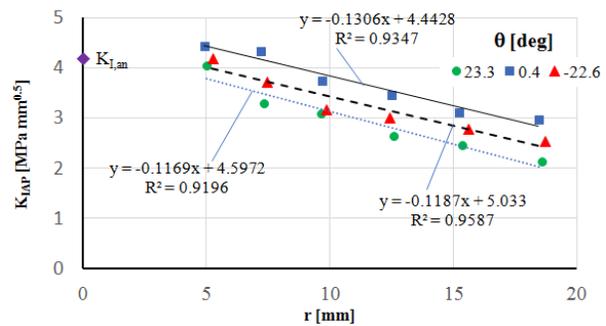


Fig. 7. SIF results based on eq. (4)

The results of the extrapolation method are shown in Fig. 7. The interrogation of displacements was performed on three directions corresponding to  $\theta = 23.3^\circ$ ,  $0.4^\circ$  and  $-22.6^\circ$ . By applying eq. (4) the apparent SIF's were calculated for 6 marks on each direction, than the results were interpolated and extrapolated at the crack tip. The obtained experimental values of  $K_I$  are 4.597 MPa mm<sup>0.5</sup> for  $\theta = 23.3^\circ$ , 4.442 MPa mm<sup>0.5</sup> for  $\theta = 0.4^\circ$ , and 5.033 MPa mm<sup>0.5</sup> for  $\theta = -22.6^\circ$ . It could be observed that all directions overestimates the value of  $K_I$ , while the smallest error 5.8% was obtained for  $\theta = 0.4^\circ$ .

## 5. Conclusions

Mark tracking technique associated with displacement correlation method represent a simple, suggestive and relatively precise methodology to estimate mode I stress intensity factors. The accuracy of the two methods are in the limit of experimental stress analysis techniques up to 15% to analytical value for the first three nodes combinations (1-2-3-4, 3-4-5-6 and 5-6-7-8) when applying eq. (2), respectively up to 10 % for two directions for extrapolation method ( $\theta = 23.3^\circ$  and  $0.4^\circ$ ).

The advantages of combining MT and DC method are: easy to employ, not need special specimen surface treatment, only creating of some marks, and equipments a Digital Camera is enough. The methodology could be applied also to mixed mode (I and II) loading. In order to increase the accuracy of the method an over deterministic method based on nonlinear least square combined with Newton-Raphson could be employed the same like in the case of strain gauges or Digital Image Correlation (Yoneyama, 2006).

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